Name: Feedback

SA402 · Dynamic and Stochastic Models Fall 2023 · Uhan

Quiz 6 – 10/16/2023

Instructions. You have 30 minutes to complete this quiz. You may use your plebe-issue TI-36X Pro calculator. You may not use any other materials.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
2	1	
3a	1	
3b	1	
3c	1	
Total		/ 70

Problem 1. The Markov Company has a manufacturing cell that processes jobs during a 12-hour shift starting at 6 a.m. and ending at 6 p.m. Jobs leave the cell according to a Poisson process with rate $\lambda = 8$ per hour.

a. If the cell has processed exactly 10 jobs by 8 a.m., what is the probability that the cell will have processed strictly more than 30 jobs by 10 a.m.?

   This is Problem 4a from the Lesson 9 Exercises.

b. What is the probability that the cell will have processed its 50th job at or before 12 p.m.?

   This is Problem 4b from the Lesson 9 Exercises.

c. When are the first 4 jobs expected to be completed? (Assume all jobs are available starting at 6 a.m.)

This is Problem 4d from the Lesson 9 Exercises.

Problem 2. Consider the arrival of customers to a doughnut shop in Downtown Annapolis. Does the independent increments property hold? Why or why not? Briefly explain.

This is Problem 6a from the Lesson 9 Exercises.

Name: Feedback

Problem 3. Patients arrive at a hospital emergency room at a rate of 2 per hour. A doctor works a 12-hour shift from 6 a.m. until 6 p.m. Assume the arrivals occur as a Poisson process.

a. Of patients admitted to the emergency room, 14% are classified as "urgent". What is the probability that the doctor will see 6 or fewer urgent patients during her shift?

This is a slightly modified version of Problem 1a from the Lesson 10 Exercises.

b. What is the expected number of urgent patients between 9 a.m. and 3 p.m.?

See Examples 2b and 3a from Lesson 10 for similar examples.

Some of you rounded your answer for this problem. However, note that this not necessary nor correct! In probability terms, "expected" means "average", and it is perfectly fine to have a fractional average of a quantity that is integral.

c. The hospital also has a walk-in clinic to handle minor problems. Patients arrive at this clinic at a rate of 4 per hour. What is the probability that the total number of patients arriving at both the emergency room and walk-in clinic from 6 a.m. to 12 noon will be 30 or more?

This is a slightly modified version of Problem 4b from the Lesson 10 Exercises.

	$X \sim \text{Poisson}(\mu)$	$X \sim \text{Exponential}(\lambda)$	$X \sim \text{Erlang}(n, \lambda)$
pmf / pdf	$p_X(a) = \begin{cases} \dfrac{e^{-\mu}\mu^a}{a!} & \text{if } a = 0, 1, 2, \ldots \\ 0 & \text{o/w} \end{cases}$	$f_X(a) = \begin{cases} \lambda e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{o/w} \end{cases}$	$f_X(a) = \begin{cases} \dfrac{\lambda(\lambda a)^{n-1} e^{-\lambda a}}{(n-1)!} & \text{if } a \geq 0 \\ 0 & \text{o/w} \end{cases}$
cdf	$F_X(a) = \sum_{k=0}^{\lfloor a \rfloor} \dfrac{e^{-\mu}\mu^k}{k!}$	$F_X(a) = \begin{cases} 1 - e^{-\lambda a} & \text{if } a \geq 0 \\ 0 & \text{o/w} \end{cases}$	$F_X(a) = \begin{cases} 1 - \sum_{k=0}^{n-1} \dfrac{e^{-\lambda a}(\lambda a)^k}{k!} & \text{if } a \geq 0 \\ 0 & \text{o/w} \end{cases}$
expected value	$E[X] = \mu$	$E[X] = \dfrac{1}{\lambda}$	$E[X] = \dfrac{n}{\lambda}$
variance	$\text{Var}(x) = \mu$	$\text{Var}(X) = \dfrac{1}{\lambda^2}$	$\text{Var}(X) = \dfrac{n}{\lambda^2}$